# AN ANALYTICAL MODEL OF SPIRAL GALAXIES

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**ABSTRACT:** We present a self-consistent analytical model of spiral galaxies embedded in dark matter halos that follows the spirit of Mo, Mao & White (MMW, 1998) disks model in hierarchical cosmogonies. Our assumptions are similar to those of MMW but differ in detail. In particular, the definition of the virial radius has been generalized and the mass-concentration relation has been updated to adapt them to the standard concordant ΛCDM cosmogony. Besides, we have considered four realistic density distributions for the host halo: a Navarro-Frenk-White (NFW) profile; a Hernquist's model; a Burkert's model; and the Sérsic's formula. We infer estimates on the baryonic disk-mass fractions and mass-to-light ratios required by all these halo profiles to describe the observed properties of the present-day disks. Our goal is to develop a tool that can predict systematically the structure (scale length) and dynamics (maximum rotational velocity) of disks for a broad range of masses. Models like this are useful, for instance, for generating the initial conditions in numerical studies of the evolution of galaxy aggregations, like groups or clusters, where one must deal with large samples of galaxies that make impractical an individualized selection of their properties.

## KEY ASSUMPTIONS

- The disk is a thin, centrifugally supported structure with an exponential surface density profile.
- The baryonic mass of a disk, $M_d$, is some fixed, universal fraction, $m_d$, of that of the halo in which is embedded.
- Halo and disk share the same specific angular momentum.
- The adiabatic contraction approximation: the halo responds adiabatically to the slow assembly of the disk, remaining spherical as it contracts.

## NEW DEFINITIONS

### VIRIAL RADIUS

We define the outer, virial radius, $R_{vir}$ of a dark halo of virial mass $M_{vir}$ as the radius within which the halo mean density is $\Delta_{vir}$ times the mean universal density at the redshift in consideration. For the family of flat cosmologies ($\Omega_m+\Omega_\Lambda=1$) the value of $\Delta_{vir}$ is approximated by (Bryan & Norman, 1998)

$$\Delta_{vir}(z) \simeq \frac{18\Pi^2+82x-39x^2}{\Omega(z)}$$

where $\Omega(z) = \Omega_0\,(1+z^3)\,/\,H(z)^2$ and $x \equiv \Omega(z) - 1$. We adopt the standard ΛCDM flat cosmological model characterized by the values ($\Omega_0$=0.3, $\Omega_{\Lambda,0}$=0.7, h=0.7).

### CONCENTRATION

Following Bullock et al. (2001), we define the cosmological concentration as the ratio

$$c \equiv R_{vir}/r_{-2}$$

with $r_{-2}$ being the radius of the ΛCDM halos at which the logarithmic slope of the density profile equals -2.

### MASS-CONCENTRATION RELATION

We approximate the behaviour of the median $M_{vir} - c$ relation for the standard concordant ΛCDM cosmology founded in Bullock's et al. (2001) simulations over the range of masses $10^{10} \le M_{vir}/(h^{-1}\ M_\odot) \le 10^{13}$ by the simple power-law empirical model :

$$c = 14.5\left[\frac{M_{vir}}{10^{12}\ h^{-1}\ M_\odot}\right]^{-0.0918}(1+z)^{-1}$$

Here we are interested in reproducing the properties galaxies observed at redshift $z \sim 0$.

## DARK MATTER PROTO-HALO DENSITY PROFILES

We have investigated the model predictions arising from four cosmologically motivated parametric mass density dark matter proto-halo profiles:

NFW $$\rho_{NFW}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^2}$$

Sérsic $$\rho_{Ser}(r) = \rho_s \exp\left[-2n\left(\frac{r}{r_s}\right)^{1/n}\right]$$

Burkert $$\rho_{Bur}(r) = \frac{\rho_s}{\left(1+\frac{r}{r_s}\right)\left[1+\left(\frac{r}{r_s}\right)^2\right]}$$

Hernquist $$\rho_{Her}(r) = \frac{\rho_s}{\frac{r}{r_s}\left(1+\frac{r}{r_s}\right)^3}$$

where $\rho_s$ and $r_s$ are, respectively, the characteristic dark matter density and scale radius of each profile, and n is a third parameter that measures the shape of the density profile, which we fix to n=6 for galaxy-sized halos (Merritt et al. 2006, and references therein). In the two panels of Fig. 1, we compare the four empirical density models (left), and the NFW and Sérsic profiles with 3 different values of n (right) for a dark halo of mass mass $M_{vir} = 10^{12}\ h^{-1}\ M_\odot$ .

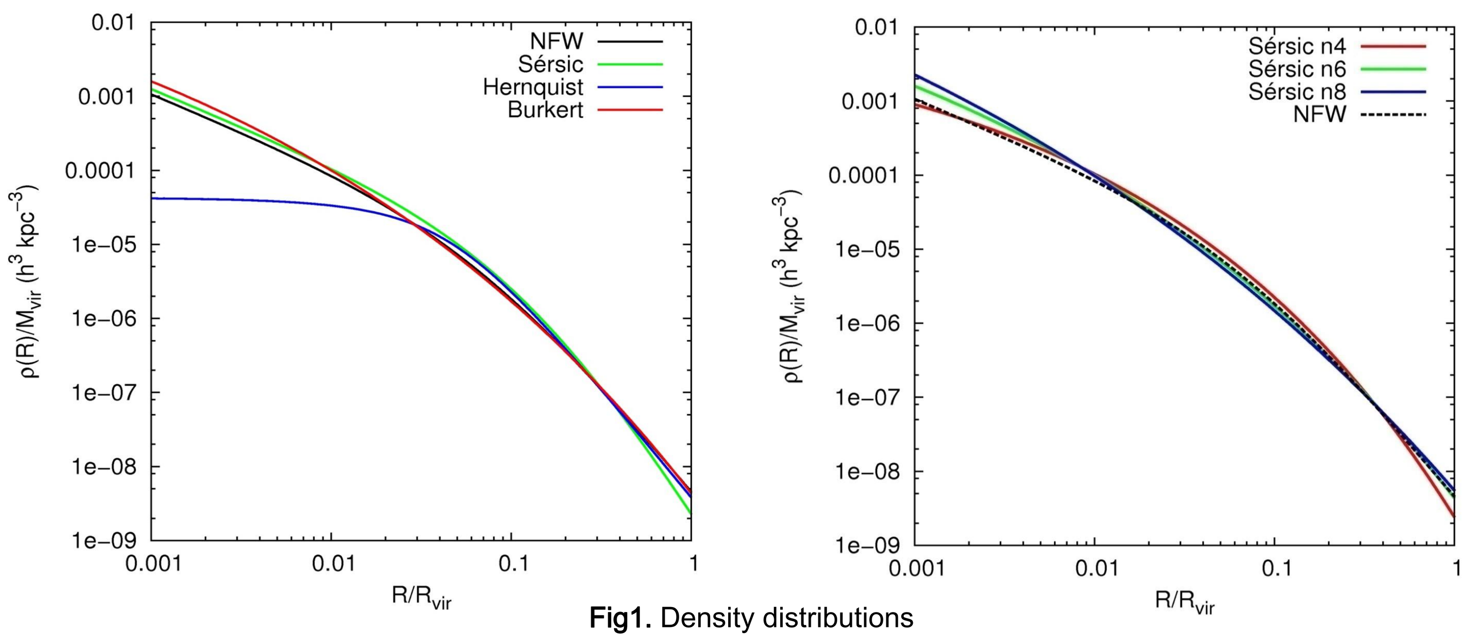


**Fig1.** Density distributions

## DISK GALAXY SCALING LAWS

### Specific angular momentum vs. maximum rotation speed

Specific angular momenta, $j_d$, are computed from disk scale lengths, $R_d$, and maximum rotation speeds, $V_{rot}$, within 5 $R_d$, assuming an exponential disk model with a flat rotation curve. Model predictions are compared against the best-fit relation from the data collected by Navarro & Steinmetz (2000) and the observational data of Vogt et al. (2004) for a sample of nearby field and cluster spirals. All galaxy properties are calculated for the I-band to limit potential confusion from active bursts of star formation. We use these data to fix the value of $m_d$ required by each halo profile (see Fig. 2 and Table 1).

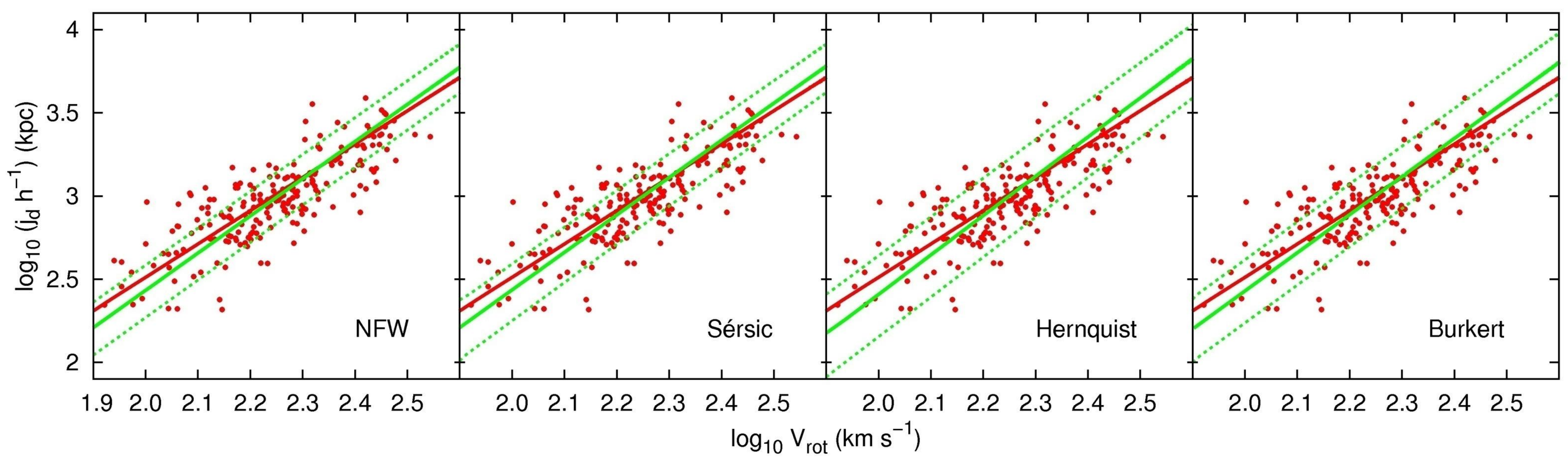


**Fig 2.** Predictions for the specific angular momentum as a function of the disk maximum rotation speed. The data points are the observational results of Vogt et al. (2004). The solid red lines represent the Navarro & Steinitz data. The green solid lines are model predictions obtained using the median c at a given total mass $M_{vir}$, while the green dashed lines encompass a 1σ variation of $\Delta\log_{10}c$ = 0.18.

### Tully-Fisher relation

We use the observed TF relation of Masters et al. (2006) to fix the stellar mass-to-light ratio of our disks, $\Upsilon_d \equiv M_d / L_d$ (Fig. 3). We list in Table 1 the values of this parameter required to match the normalization of this relationship for each proto-halo profile. Considering $m_d \propto M_{vir}^{\beta}$, it is possible to match both the normalization and the slope of the observed TF relation. We obtain $\beta \ge 0.2$, which implies a baryonic TF slope of ~ 3.5 or larger.

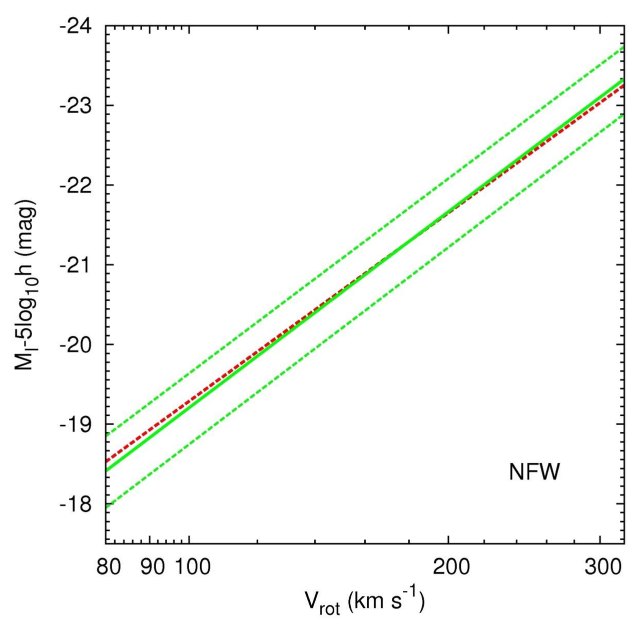


**Fig 3.** Tully-Fisher relation. The dashed red line shows the observed TF relation of Masters et al. (2006). Green lines are the model predictions for NFW proto-halos. The other halo profiles show a similar behavior.

### Central disk surface density vs. maximum rotation speed

Once $m_d$ and $\Upsilon_d$ have been fixed, our model predictions provide a fairly good match to the observations of this relationship (see Fig. 4).

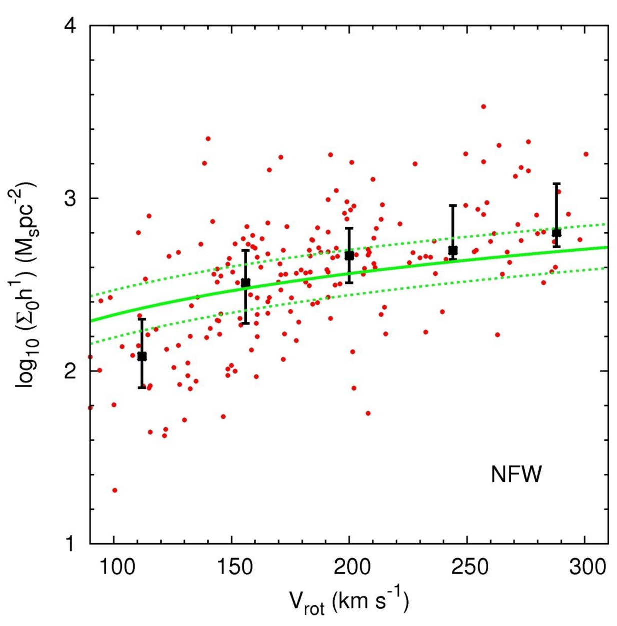


**Fig 4.** Central disk surface density vs. maximum rotation speed. Red points are the observational data by Vogt et al. (2004). Black squares are plotted at the median value in each bin and the ends of the bars mark the first and third quartiles. Green lines are the model predictions for the NFW proto-halos.

| | $m_d$ | $\Upsilon_d$ |
|---|---|---|
| NFW | 0.04 | 1.15h |
| Sersic | 0.04 | 1.15h |
| Hernquist | 0.005 | 0.20h |
| Burkert | 0.03 | 0.90h |

**Table 1.** Values of $m_d$ and $\Upsilon_d$ .

## ROTATION CURVES

Here, we compare the predicted rotation curves of disks formed in halos with an initial density distribution described by the NFW, Sérsic, Hernquist, and Burkert profiles corresponding to the values of $m_d$ listed in Table 1 for a galactic halo of total mass $M_{vir} = 10^{12}\ h^{-1}\ M_\odot$. We also show the Milky Way (MW) value of the rotational curve at the solar radius.

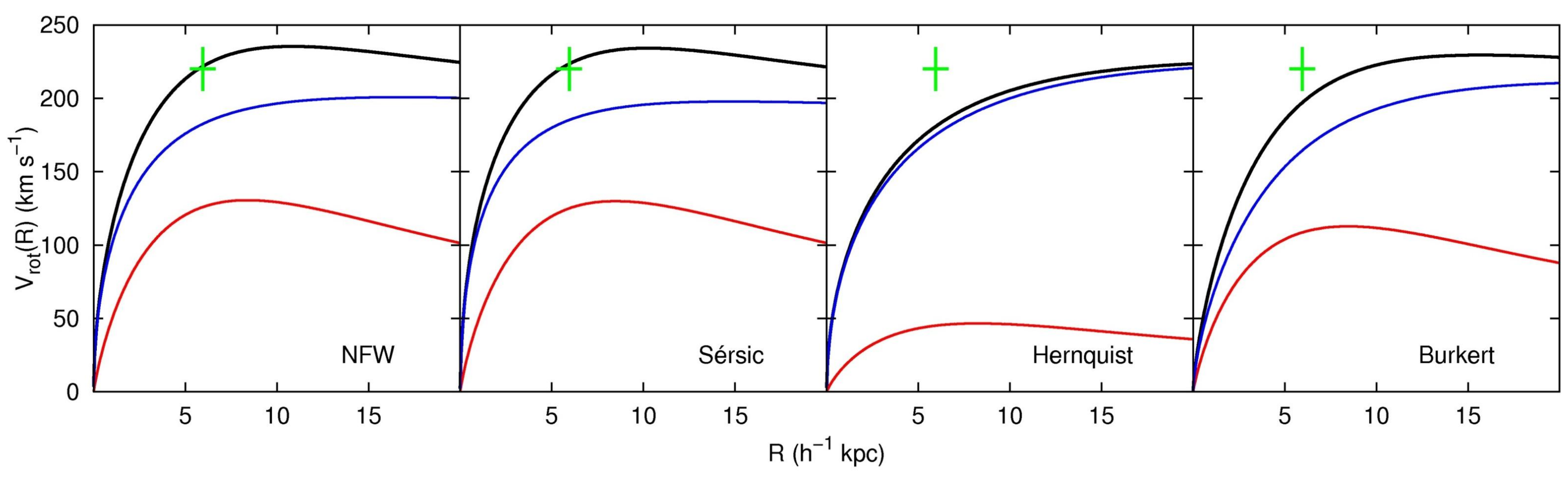


**Fig 5.** Rotation curves. The black lines show the total rotation velocity, while the rotation velocities induced by the disk and the dark matter are shown with blue and red lines respectively. The green crosses represent the range of values of the MW solar neighbourhood, R = 8.5 ± 1 kpc and $V_{rot}$ = 220 ± 15 km/s.

## CONCLUSIONS

- The extremely low values of $m_d$ required for Hernquist halos in order to match the normalization of the $j_d - V_{rot}$ relation rule out this profile as a suitable description of the proto-halo mass distribution of our disk-galaxy model. In contrast, the NFW, Burkert, and Sérsic (n = 6) halo profiles require values of $m_d$ consistent with expectations.
- For all the halo profiles investigated the slopes of the predicted TF relations are fully consistent with the observational data. However, while the NFW and Sérsic profiles need a stellar mass-to-light ratio of about 1.2 h in order to match the normalization of this relationship, the Hernquist and Burkert profiles require values of $\Upsilon_d$ that are too low compared with observational estimates.
- The NFW and Sérsic (n = 6) with $m_d$ = 0.04 not only reproduce satisfactorily all the scaling relations investigated, but also produce rotation curves for $M_{vir} = 10^{12}\ h^{-1}\ M_\odot$ that match the values of the rotation curve of the Milky Way at the solar radius.

## REFERENCES

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